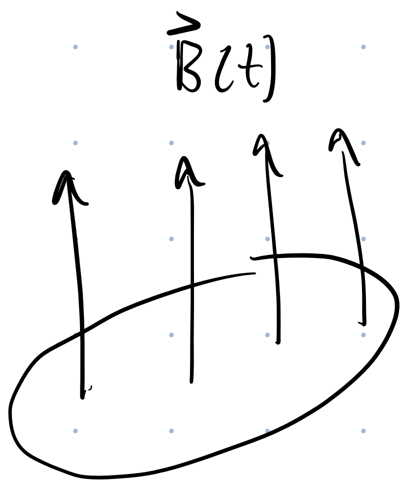
PHYS 301 Tutorial #9 Group Prob. Sol'ns Nov. 25, 2024

1. 

Faraday's Law: $\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\frac{\partial B}{\partial t}\hat{z}$

$\therefore$ need z-component of the $\vec{\nabla}\times\vec{E}$ in cylindrical coordinates.

$$\frac{1}{s}\left[\frac{\partial}{\partial s}\left(s E_\phi\right) - \frac{\partial E_s}{\partial \phi}\right] = -\frac{\partial B}{\partial t}$$

From RHR, expect $\vec{E} = E_\phi \hat{\phi}$

$$\therefore \frac{\partial}{\partial s}\left(s E_\phi\right) = -s\frac{\partial B}{\partial t}$$

$$\therefore s E_\phi = -\int s \frac{\partial B}{\partial t} ds$$

since $\vec{B}$ is uniform:

$$s E_\phi = -\frac{\partial B}{\partial t}\int s\,ds = -\frac{s^2}{2}\frac{\partial B}{\partial t}$$

$$\therefore E_\phi = -\frac{s}{2}\frac{\partial B}{\partial t}, \quad \text{finally} \quad \boxed{\vec{E} = -\frac{s}{2}\frac{\partial B}{\partial t}\hat{\phi}}$$

Alternative approach:

Again, can start w/ Faraday's Law

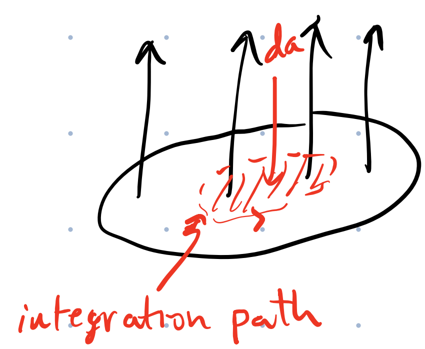
$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$ integrate over a surface ...

Stoke's law

$$\underbrace{\oint \vec{\nabla}\times\vec{E}\cdot d\vec{a}}_{\oint \vec{E}\cdot d\vec{l}} = -\frac{\partial}{\partial t}\underbrace{\int \vec{B}\cdot d\vec{a}}_{\Phi}$$

$$\therefore \oint \vec{E}\cdot d\vec{l} = -\frac{\partial \Phi}{\partial t}$$

In this problem, $\Phi = \vec{B}(t)\,\pi s^2$

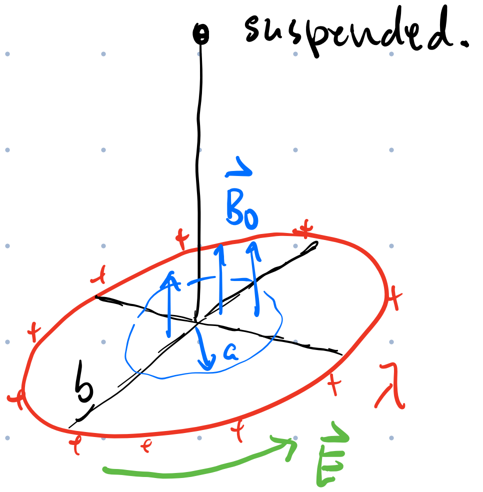


$$\therefore \quad E\, 2\cancel{\pi} \cancel{s} = -\frac{\partial B}{\partial t} \cancel{\pi} s^{\cancel{2}}$$

$$\therefore E = -\frac{s}{2}\frac{\partial B}{\partial t} \Rightarrow \boxed{\vec{E} = -\frac{s}{2}\frac{\partial B}{\partial t}\hat{\phi}}$$

same as before.

2.



$$\Phi_i = \pi a^2 B_0 \qquad \Phi_f = 0$$

$$\therefore \mathcal{E} = -\frac{\Delta \Phi}{\Delta t} = \frac{\pi a^2 B_0}{\Delta t}$$

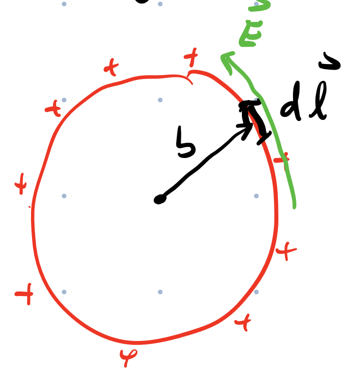
$$\mathcal{E} = \oint \vec{E}\cdot d\vec{l} = E\,2\cancel{\pi} b = \frac{\cancel{\pi} a^2 B_0}{\Delta t}$$

loop of
radius b

$$\therefore \vec{E} = \frac{a^2 B_0}{2b\Delta t}\,\hat{\phi}$$

Find the resulting torque on charged loop.

Top view



$$dq = \lambda\, dl$$
$$= \lambda b\, d\phi$$

Torque on element of length dl is:

$$d\vec{N} = b\hat{s} \times dq\,\vec{E}$$

$$= b\hat{s} \times \lambda b\, d\phi\, \frac{a^2 B_0}{2b\Delta t}\hat{\phi}$$

$$= \frac{\lambda a^2 b^2 B_0\, d\phi}{2b\,\Delta t}\underbrace{(\hat{s} \times \hat{\phi})}_{\hat{z}}$$

$$= \frac{\lambda a^2 b\, B_0\, d\phi}{2\Delta t}\hat{z}$$

$$\therefore \vec{N} = \frac{\lambda a^2 b B_0}{2\Delta t}\underbrace{\int d\phi}_{2\pi}\,\hat{z} = \frac{\pi \lambda a^2 b B_0}{\Delta t}\hat{z}$$

$$\vec{N} = \frac{d\vec{L}}{dt} \Rightarrow \Delta\vec{L} = \vec{N}\Delta t$$

but $\vec{L}_i = 0$

angular momentum $\therefore \vec{L} = \vec{N}\Delta t = \pi \lambda a^2 b B_0 \hat{z}$

Finally $\vec{L} = I_{rot}\,\vec{\omega}$

$$\therefore\ mb^2\vec{\omega} = \pi\lambda a^2 b B_0\,\hat{z}$$

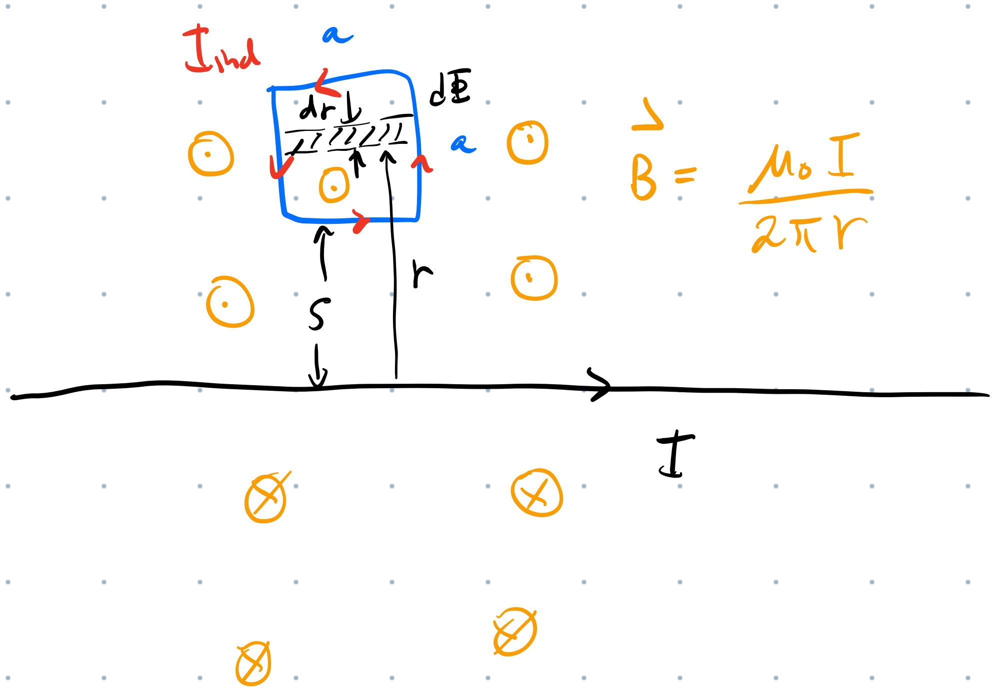
$$\boxed{\therefore\ \vec{\omega} = \frac{\pi\lambda a^2 B_0}{mb}\,\hat{z}}$$

check units $[\omega] = \frac{C}{\cancel{m}}\,\frac{\cancel{m^2}\,T}{kg\,\cancel{m}}$

$$= \frac{C}{\cancel{kg}}\,\frac{\cancel{kg}}{s^2 A}$$

$$= \frac{\cancel{C}}{\cancel{s}}\,\frac{1}{\cancel{A}}\,\frac{1}{s} = \frac{1}{s}\ \checkmark$$

3.



$$d\Phi = B(r)\,a\,dr = \frac{\mu_0 I a}{2\pi r}\,dr$$

$$\therefore \Phi = \int_{r=s}^{s+a} d\Phi = \frac{\mu_0 I a}{2\pi}\int_s^{s+a}\frac{dr}{r} = \frac{\mu_0 I a}{2\pi}\ln\left(1+\frac{a}{s}\right)$$

$$\therefore \mathcal{E} = -\frac{d\Phi}{dt} = -\frac{\mu_0 a}{2\pi}\ln\left(1+\frac{a}{s}\right)\frac{dI}{dt}$$

$$\frac{dI}{dt} = \frac{d}{dt}\left[I_0(1-\alpha t)\right] = -\alpha I_0$$

$$\therefore I_{ind} = \frac{\mathcal{E}}{R} = \frac{\mu_0 a \alpha I_0}{2\pi R}\ln\left(1+\frac{a}{s}\right)$$

I_{ind} opposes that decreasing Φ. $\therefore$ $\boxed{I_{ind} \text{ is ccw}}$

$$I_{ind} = \frac{dq}{dt}$$

time to turn current to zero.

$$\therefore q = \int_0^{1/\alpha} I_{ind}\, dt$$

$$= \int_0^{1/\alpha} \underbrace{\frac{\mu_0 a \alpha I_0}{2\pi R} \ln\left(1 + \frac{a}{s}\right)}_{\text{all constants.}} dt$$

$$\boxed{\therefore q = \frac{\mu_0 a I_0}{2\pi R} \ln\left(1 + \frac{a}{s}\right)}$$

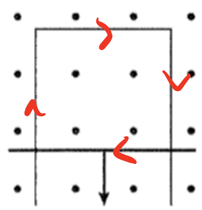
check units: $[q] = \frac{\cancel{N}}{\cancel{A^2}} \cancel{m}\, A \; \frac{s \cancel{A^2}}{\cancel{N}\cancel{m}} = \frac{C}{s} s = C$ ✓

4.

The figures below show one or more metal wires sliding on fixed metal rails in a magnetic field. For each, determine if the induced current flows clockwise, flows counterclockwise, or is zero. Show your answer by drawing it.

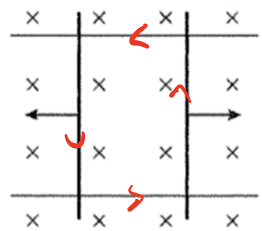
a.

Φ increasing out of page. $\therefore I_{ind}$ cw



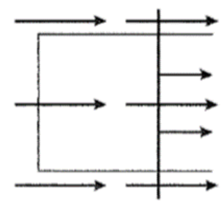
b.

$\Phi \uparrow$ into page $\therefore$



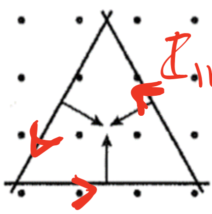
I_{ind} ccw

c.



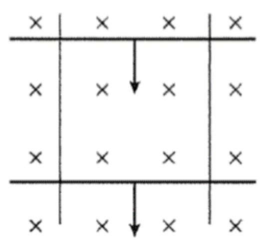
$\vec{a} \perp \vec{B}$ $\therefore \Phi = 0$ & const. $\therefore I_{ind} = 0$

d.



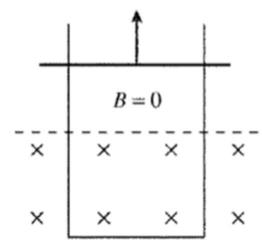
$\Phi \downarrow$ out of page. $\therefore I_{ind}$ ccw

e.



$\Phi = $ const. $\therefore I_{ind} = 0$

f.

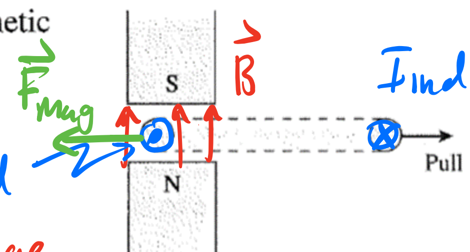


$\Phi = $ const. $\therefore I_{ind} = 0$

5.

A loop of copper wire is being pulled from between two magnetic poles.

a. Show on the figure the current induced in the loop. Explain your reasoning.



Φ decreasing towards top of page.

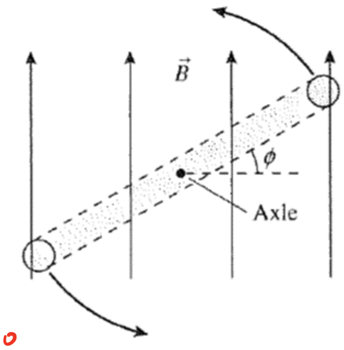
$\therefore I_{ind}$ creates $\vec{B}_{ind}$ towards top of page.

b. Does either side of the loop experience a magnetic force? If so, draw and label a vector arrow or arrows on the figure to show any forces.

The left side of the loop experiences a force.

$\vec{F}_{mag} \propto \vec{I} \times \vec{B}$ $\therefore \vec{F}_{mag}$ to the left

6.



A circular loop rotates at constant speed about an axle through the center of the loop. The figure shows an edge view and defines the angle ϕ, which increases from 0° to 360° as the loop rotates.

a. At what angle or angles is the magnetic flux a maximum?

Φ is max when $\phi = 0$ & $180°$

b. At what angle or angles is the magnetic flux a minimum?

Φ is min (equal to zero) when $\phi = 90°, 270°$

c. At what angle or angles is the magnetic flux *changing* most rapidly? Explain your choice.

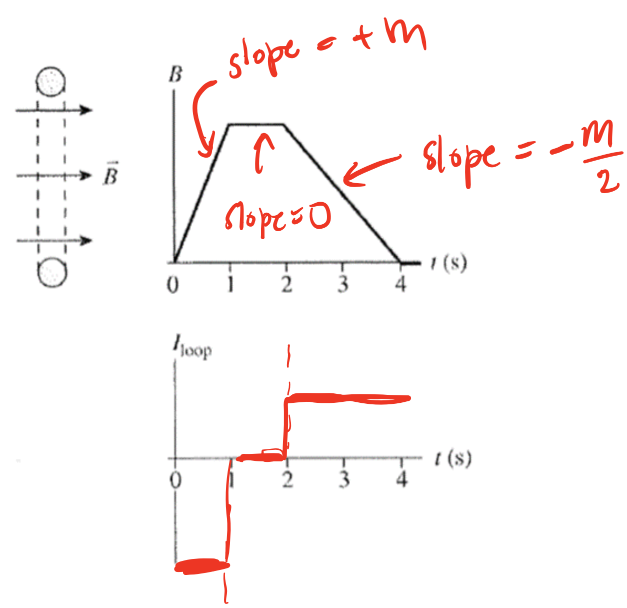
$$\Phi = \vec{B} \cdot \vec{a} = B a \cos\phi$$

$$\frac{d\Phi}{dt} = -B a \omega \sin\phi \qquad \therefore \frac{d\Phi}{dt} \text{ is max when } \phi = 90° \text{ \& } 270°$$

$d\phi/dt$

7.

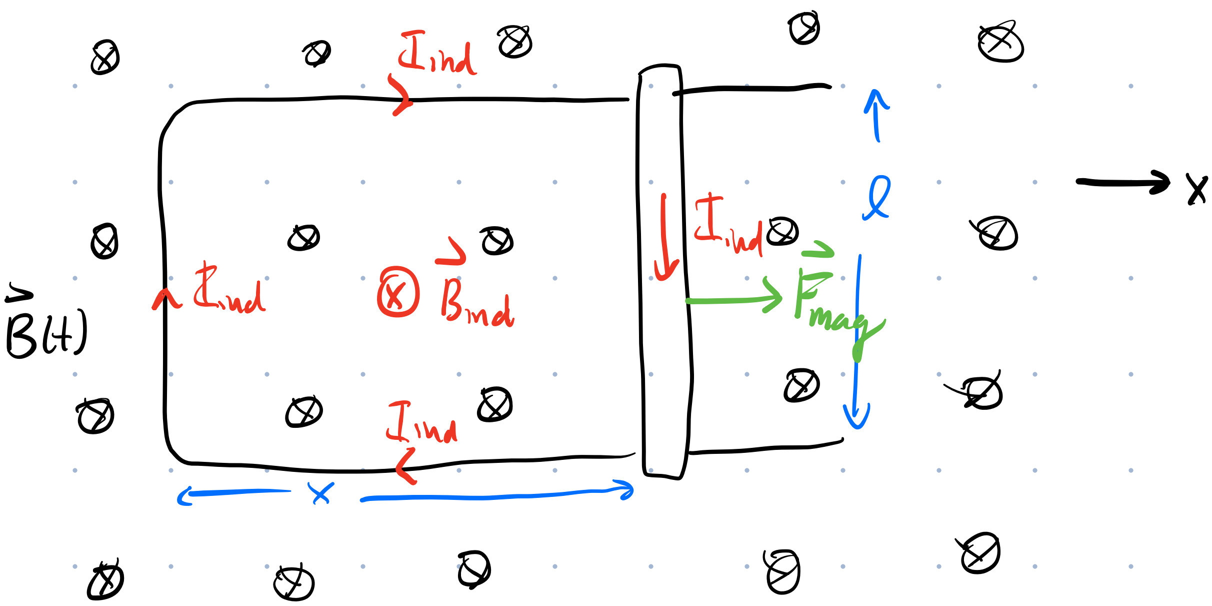
A loop of wire is perpendicular to a magnetic field. The magnetic field strength as a function of time is given by the top graph. Draw a graph of the current in the loop as a function of time. Let a positive current represent a current that comes out of the top and enters the bottom. There are no numbers for the vertical axis, but your graph should have the correct shape and proportions.



$$I_{loop} = \frac{\mathcal{E}}{R} = -\frac{1}{R}\frac{d\Phi}{dt}$$

$$\therefore I_{loop} \propto -\text{slope}$$

8.



$$\Phi = Bx\ell$$

$$\mathcal{E} = -\frac{\partial \Phi}{\partial t} = -x\ell \frac{\partial B}{\partial t}$$

$$\therefore I_{ind} = -\frac{x\ell}{R}\frac{\partial B}{\partial t}$$

b/c $\vec{B}\downarrow$, I_{ind} creates $\vec{B}_{ind}$ to oppose the change in Φ,

i.e. $\vec{B}_{ind}$ into page & $\boxed{I_{ind} \text{ is CW}}$

$$\vec{F}_{mag} = \vec{I}_{ind} \times \vec{B}\ell = I_{ind} B \ell \hat{x}$$

$$\therefore \vec{F}_{mag} = m\vec{a} = \frac{x\ell}{R} B\ell \frac{\partial B}{\partial t}\hat{x} = \frac{x\ell^2 B}{R}\frac{\partial B}{\partial t}\hat{x}$$

$$\boxed{\therefore\ \vec{a} = \frac{x l^2 B}{mR}\frac{\partial B}{\partial t}\,\hat{x}}$$

Bar accelerates to the right.

check units:

$$[a] = \frac{m\ m^2\ T}{kg\ \Omega}\ \frac{T}{s}$$

$$= \frac{\cancelto{2}{m^3}\ T^2}{kg\ \cancel{s}}\ \frac{\cancelto{2}{s^3}\ A^2}{kg\ \cancel{m^2}}$$

$$= \frac{m\ T^2 s^2 A^2}{kg^2} = \frac{m\ \cancel{s^2}\ \cancel{A^2}}{\cancel{kg^2}}\left(\frac{\cancel{kg}}{\cancel{s^2}\ \cancel{A}}\right)^2$$

$$= \frac{m}{s^2}\ \checkmark$$